METHOD OF PROOF

TYPES OF METHOD OF PROOF

1. Direct proof .
2. Indirect proof.

Indirect proof there will be two methods.

- Contrapositive method.
- Contradictive method.

3. Counter example.

Basic Definitions

An integer n is an even number
if there exists an integer k such that $n = 2k$.

An integer n is an odd number
if there exists an integer k such that $n = 2k+1$.

DIRECT PROOF

- In direct proof we assume the given condition p and proof the result directly.

Direct proof:

- An implication $p \rightarrow q$ can be proved by showing that if p is true, then q is also true.

EXAMPLE.

EXERCISE:

Prove that the product of an even integer and an odd integer is even.

SOLUTION:

Suppose m is an even integer and n is an odd integer. Then

$m = 2k$ for some integer k

and $n = 2l + 1$ for some integer l

Now

$$m \cdot n = 2k \cdot (2l + 1)$$
$$= 2 \cdot k\,(2l + 1)$$
$$= 2 \cdot r \quad \text{where } r = k(2l + 1) \text{ is an integer}$$

Hence m·n is even. (Proved)

EXAMPLE.

The product of two odd numbers is odd .

Proof.

$X=2m+1$, $Y=2n+1$

$XY=(2m+1)(2n+1)$

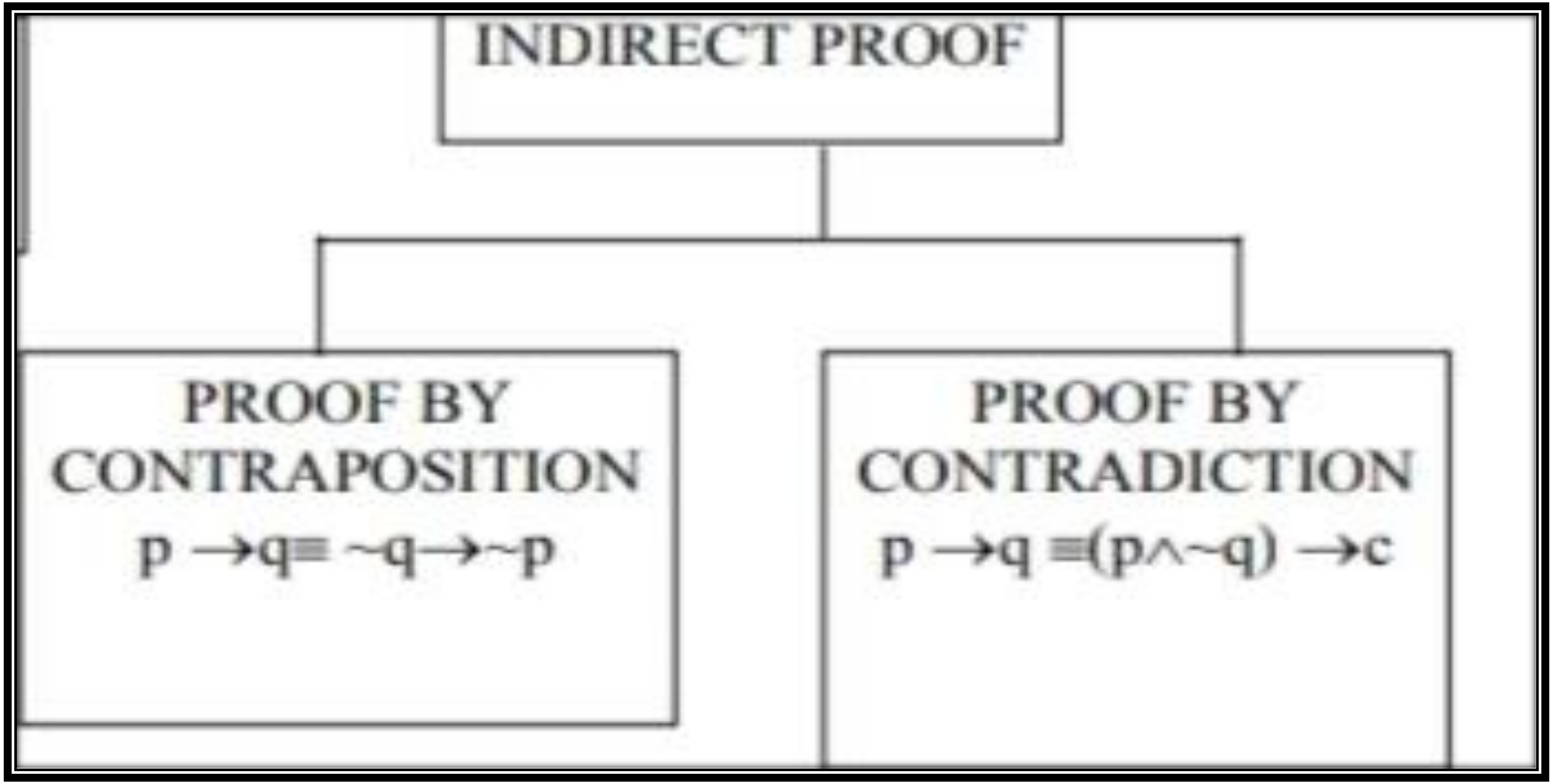
$=4mn+2m+2n+1$

$=2(2mn+n+m)+1$

$2mn+n+m=k$

$XY=2k+1$. That's odd

INDIRECT PROOF.



CONTRAPOSITION METHOD.

The method of proof by contrapositive maybe summarize.

- Express the statement in the form of if p then q.
- Rewrite this statement in the contrapositive form.

  If not q then not p

- Prove the contrapositive by direct debit.

EXAMPLE.

Proof the Contrapositive

For an integer n, n is even if and only if n^2 is even.

Method 1b: Prove P implies Q and not P implies not Q.

Statement: If n^2 is even, then n is even

Contrapositive: If n is odd, then n^2 is odd.

Proof (the contrapositive):

Since n is an odd number, $n = 2k+1$ for some integer k.

So $n^2 = (2k+1)^2$

$= (2k)^2 + 2(2k) + 1 \quad = 2(2k^2 + 2k) + 1$

EXAMPLE 2.

Prove that if $(n)^2$ is not divisible by 25 then n is not divisible by 5.

Proof.

The contrapositive statement is

if n is divisible by 5 , then $(n)^2$ is divisible by 25

Suppose n is divisible by 5

$n = 5k$. For some integer k

Squaring on both sides

$(n)^2 = (5k)^2$

$n^2 = 25k^2$ where $k^2 € Z$

n^2 is divisible by 25.

CONTRADICTIONS METHOD.

- In this method,

We assume the opposite of what we are trying to prove and get logical contradictions Hence our

Assumption is wrong. Then the original result must be true.

EXAMPLE.

Suppose n €Z

If 3n+2 is odd, then n is odd.

Solution.

According to contradictions,

Let n is even

Then n=2k. K€Z

Now, 3n+2=3(2k)+2

6k+2= 2(3k+1)

2m. Where 3k+1 =m

- 2m is even

our assumption is wrong

Hence n is odd.

COUNTER EXAMPLE

1. In this method we can prove it disprove the statement by taking example.
2. Prove or disprove the statement if X and Y are real numbers.

EXAMPLE 1.

- Prove or disprove the statement if X and Y are real numbers .

$X^2=Y^2$.   if and only if. $X=Y$

Solution

if we take two numbers

-3 and 3

$(-3)^2=(3)$

9=9   but -3 and 3 are not equal

So the statement is false

THE END