

BS CS

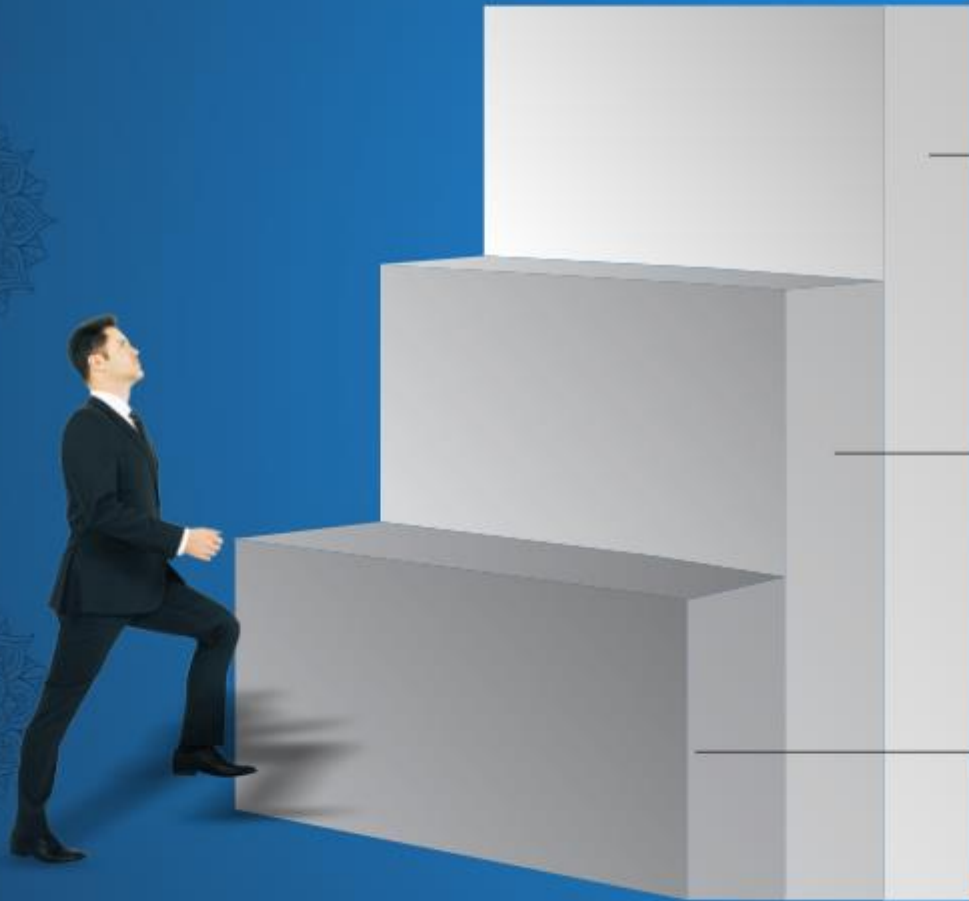
DISCRETE MATHEMATICS

Topic:

FUNCTIONS

Introduction to Functions and Relations

Understanding Functions and Relations in Discrete Mathematics



Definition of Functions and Relations

1

Functions and relations explain how elements from one set interact with elements from another set, forming crucial mathematical relationships.

Significance in Mathematical Modeling

2

These concepts are essential for building models that simulate real-world situations in various fields including engineering and economics.

Applications in Computer Science

3

Functions and relations are widely used in algorithms, data structures, and database management, making them integral to computer science.

Understanding Functions

A relation f from a set X to Y ,
is called *function*, denoted
 $f: X \longrightarrow Y$, is a relation from X , the
domain, to Y , the *co-domain*.

Have you thought how a **relation** becomes a **function**?

- ✓ Domain of Relation **R** is = **X** (First Set)
- ✓ There is no repetition in the first elements of **Relation**.



Let's solve an example:

$$X = \{2, 4, 6\}$$

$$Y = \{1, 3, 5, 7\}$$

Let's check the following relation is a function:

➤ $R1 = \{ (2, 1), (2, 3), (2, 5), (4, 7) \}$

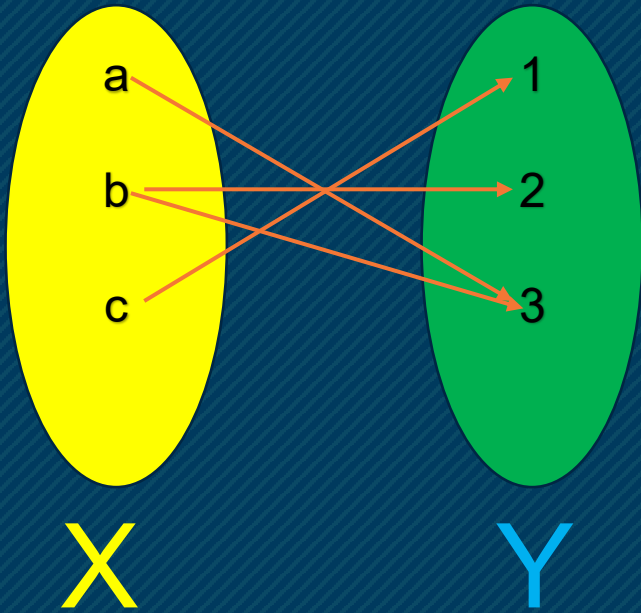
➤ $R2 = \{ (4, 5), (6, 5), (6, 7) \}$

➤ $R3 = \{ (2, 3), (4, 5), (6, 7) \}$

- ❖ $R1$ is not a function, because $6 \in X$ does not appear as first element in any ordered pair in $R1$.
- ❖ $R2$ is not a function, because the ordered pairs $(6, 5)$ and $(6, 7)$ have the first element and $2 \in X$ does not appear as the first element in ordered pair in $R2$.
- ❖ $R3$ defines a function.

Let's understand this from a diagram:

A.

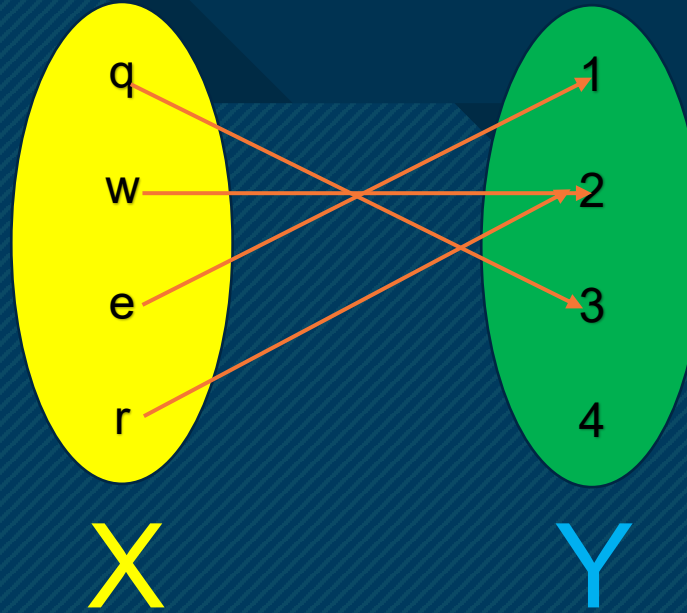


X

Y

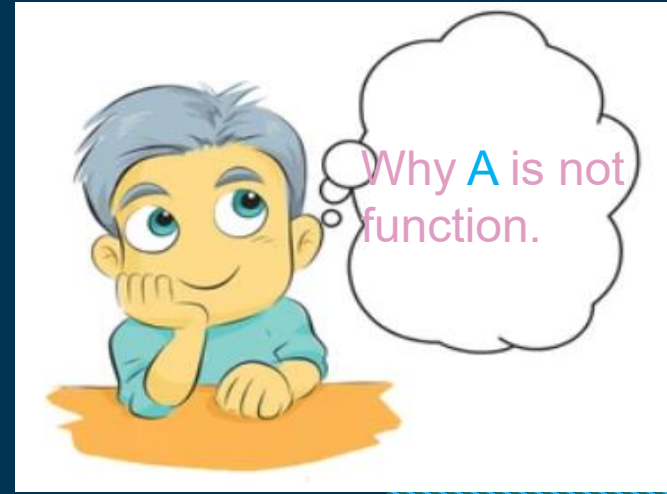
B is a function

B.



X

Y



Types of

Function

Injective Function

OR

Into

Surjective

OR

Onto

Bijjective Function

AND

One-to-One
Function

One-to-One (1-1):

DEFINITION:

f is **one-to-one** if it takes distinct points of the domain to distinct points of the **co-domain**.

Example:

$$X = \{2, 4, 5\}$$

$$Y = \{1, 2, 4, 6\}$$

$$X \times Y =$$

$$\{(2, 1), (2, 2), (2, 4), (2, 6), (4, 1), (4, 2), (4, 4), (4, 6), (5, 1), (5, 4), (5, 6)\}$$

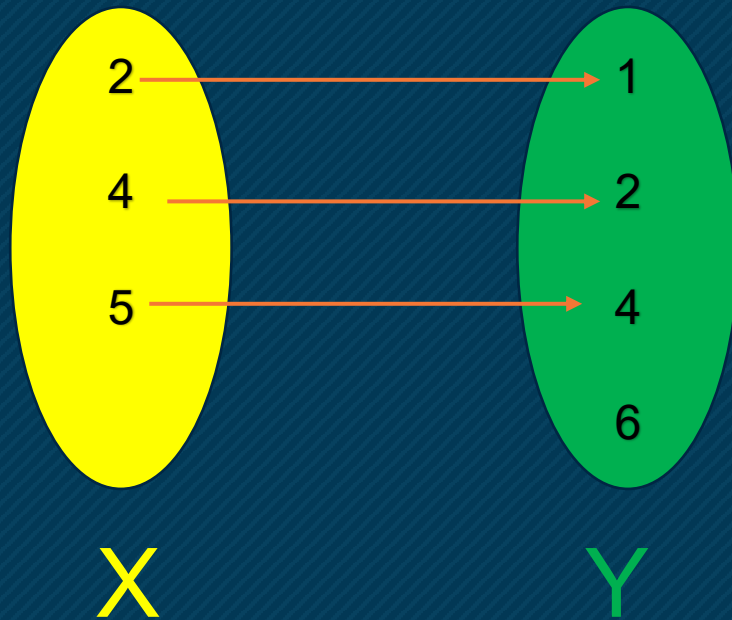
$$R = \{(2, 1), (4, 2), (5, 4)\}$$

$$\text{Domain of } R = \{2, 4, 5\}$$

$$\text{Range of } R = \{1, 2, 4\}$$

As you see Domain of $R = X$ and first elements of R doesn't repeat .

So, R is a function.



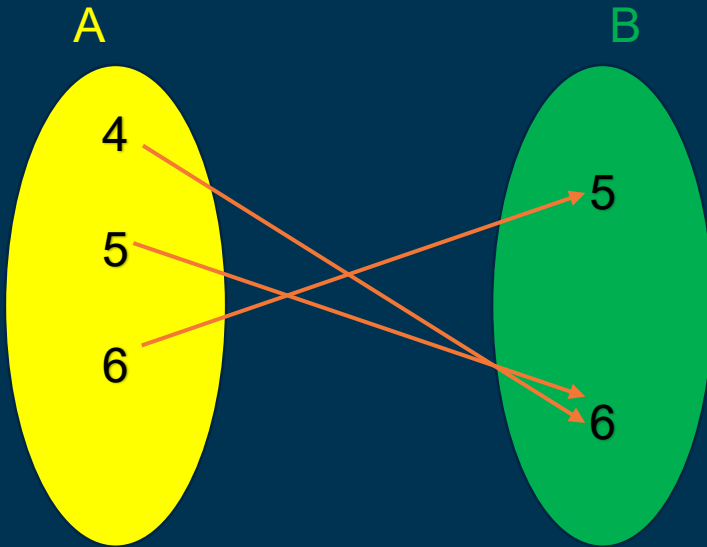
Range of R is a proper subset of Y
Range $R \subset Y$. So, it is an
Into/injective function.
It is also one-to-one function.

SURJECTIVE/ONTO:

DEFINITION:

A function $f:A \rightarrow B$ is **surjective (onto)** if every element in B is the image of at least one element in A . This means the **range** of f is equal to the **codomain** of B .

Example:



$$A = \{4, 5, 6\}$$

$$B = \{5, 6\}$$

F is a relation from $A \times B$

$$R = \{(4, 6), (5, 5), (6, 6)\}$$

Range of $R = B$

As you see first elements of R doesn't repeat. So, R is a function.

Range = B

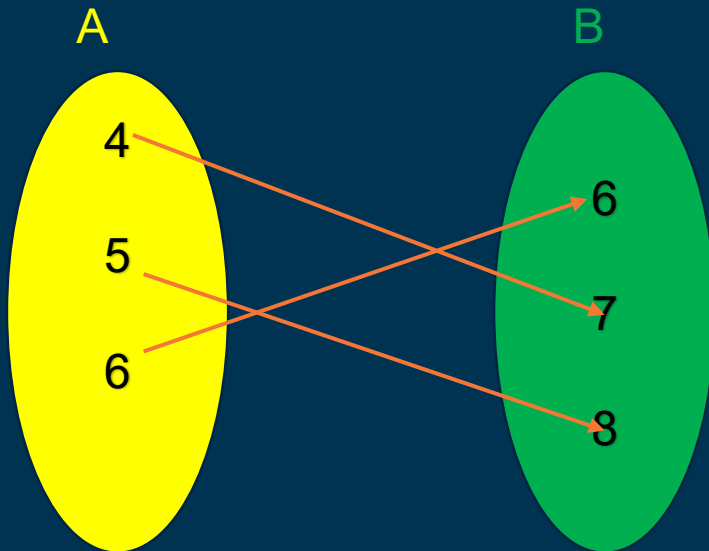
So, it is an **onto function**.

BIJECTIVE FUNCTION:

A function $f:A \rightarrow B$: is **bijective** if it is both **one-to-one** and **surjective**.

This means every **element** in **A** maps to a **unique element** in **B**, and every **element** in **B** is **covered**.

EXAMPLE:



$$A = \{4, 5, 6\}$$

$$B = \{6, 7, 8\}$$

R is relation from $A \times B$

$$R = \{ (4, 6), (5, 7), (6, 8) \}$$

As you see Domain of $R=A$ and first elements of R doesn't repeat . So, R is a function.

Range of $R = B$ (onto) and also there is **one to one** linkage.

So, it's a **Bijective function**.

THANKS FOR YOUR TIME.

